

Observational Constraints on Teleparallel Dark Energy

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ABSTRACT: We use data from Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO), and Cosmic Microwave Background (CMB) observations to constrain the recently proposed teleparallel dark energy scenario based on the teleparallel equivalent of General Relativity, in which one adds a canonical scalar field, allowing also for a nonminimal coupling with gravity. Using the power-law, the exponential and the inverse hyperbolic cosine potential ansatzes, we show that the scenario is compatible with observations. In particular, the data favor a nonminimal coupling, and although the scalar field is canonical the model can describe both the quintessence and phantom regimes.

KEYWORDS: Modified gravity, dark energy, phantom-divide crossing, $f(T)$ gravity, non-minimal coupling

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1 Introduction.

Recent cosmological observations [1–5] indicate that the observable universe experiences an accelerated expansion. Although the simplest way is the consideration of a cosmological constant, there are two alternative ways to explain this behavior. The first is to modify the content of the universe by introducing the dark energy sector, which can be based on a canonical scalar field (quintessence) [6–22], a phantom field [23–28], or on the combination of both these fields in a unified scenario called quintom [29–35] (see [36] for a review). The other approach is to modify the gravitational sector itself (see [37] for a review and references therein).

The *teleparallel* dark energy is a recently proposed scenario that tries to incorporate the dark energy sector [38]. It is based on the “teleparallel” equivalent of General Relativity (TEGR), that is on its torsion instead of curvature formulation [39, 40], in which one adds a canonical scalar field, and the dark energy sector is attributed to this field. In the case where the field is minimally coupled to gravity the scenario is completely equivalent to the standard quintessence [6–11], both at the background and perturbation levels. However, in the case where one switches on the nonminimal coupling between the field and the torsion scalar, that is the only suitable gravitational scalar in TEGR, the resulting scenario has a richer structure, exhibiting quintessence-like or phantom-like behavior, or experiencing the phantom-divide crossing [38]. The richer structure is manifested in the absence of a conformal transformation to an equivalent minimally-coupled model with transformed field and potential, which is known to be able to describe only the quintessence regime.

Since the teleparallel dark energy exhibits interesting cosmological behavior, in the present work we use observations in order to constrain the parameters of the model. In particular, we use data from Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO), and Cosmic Microwave Background (CMB) to plot likelihood-contours for the present dark-energy equation-of-state, matter density parameter and nonminimal coupling parameters, respectively.

The paper is organized as follows: In Section 2 we present the scenario of the teleparallel dark energy and derive the relevant equations for the cosmological evolution. In Section 3 we use observational data to produce likelihood-contours of the model parameters. Finally, Section 4 is devoted to the summary of our results.

2 Teleparallel Dark Energy

Let us briefly review the teleparallel dark energy. As we stated in Introduction, it is based on the “teleparallel” equivalent of General Relativity (TEGR) [39, 40], in which instead of using the torsionless Levi-Civita connection one uses the curvatureless Weitzenböck one. The dynamical objects are the four linearly independent vierbeins (these are parallel vector fields, referred to as the appellations “teleparallel” or “absolute parallelism”). It is interesting to note that the torsion tensor is formed solely from products of the first derivatives of the tetrad.

In particular, the vierbein field $\mathbf{e}_A(x^\mu)$ forms an orthonormal basis for the tangent space at each point x^μ , that is $\mathbf{e}_A \cdot \mathbf{e}_B = \eta_{AB}$, where $\eta_{AB} = \text{diag}(1, -1, -1, -1)$, and furthermore the vector \mathbf{e}_A can be analyzed with the use of its components e^μ_A in a coordinate basis, that is $\mathbf{e}_A = e^\mu_A \partial_\mu$ ¹. In such a construction, the metric tensor is obtained from the dual vierbein as

$$g_{\mu\nu}(x) = \eta_{AB} e^\mu_A(x) e^\nu_B(x). \quad (2.1)$$

Consequently, the torsion tensor of the Weitzenböck connection $\overset{\mathbf{w}}{\Gamma}_{\nu\mu}^\lambda$ [43] reads

$$T_{\mu\nu}^\lambda = \overset{\mathbf{w}}{\Gamma}_{\nu\mu}^\lambda - \overset{\mathbf{w}}{\Gamma}_{\mu\nu}^\lambda = e^\lambda_A (\partial_\mu e_\nu^A - \partial_\nu e_\mu^A), \quad (2.2)$$

where $\overset{\mathbf{w}}{\Gamma}_{\nu\mu}^\lambda \equiv e^\lambda_A \partial_\mu e_\nu^A$.

In the present formalism, all the information concerning the gravitational field is included in the torsion tensor $T_{\mu\nu}^\lambda$. As described in [40], the “teleparallel Lagrangian” can be constructed from this torsion tensor under the assumptions of invariance under general coordinate transformations, global Lorentz transformations, and the parity operation, along with requiring the Lagrangian density to be the second order in the torsion tensor. In particular, it is the torsion scalar T , given by [39, 40, 44, 45]:

$$\mathcal{L} = T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu. \quad (2.3)$$

The simplest action in a universe governed by teleparallel gravity is

$$S = \int d^4x e \left[\frac{T}{2\kappa^2} + \mathcal{L}_m \right], \quad (2.4)$$

where $e = \det(e_\mu^A) = \sqrt{-g}$ (one could also include a cosmological constant). Variation with respect to the vierbein fields provides equations of motion, which are exactly the same as

¹We follow the notation of [41, 42], that is Greek indices μ, ν, \dots and capital Latin indices A, B, \dots run over all coordinate and tangent space-time 0, 1, 2, 3, while lower case Latin indices (from the middle of the alphabet) i, j, \dots and lower case Latin indices (from the beginning of the alphabet) a, b, \dots run over spatial and tangent space coordinates 1, 2, 3, respectively. Finally, we use the metric signature $(+, -, -, -)$.

those of GR for every geometry choice, and that is why the theory is called “teleparallel equivalent to General Relativity”.

In principle one has two ways of generalizing the action (2.4), inspired by the corresponding procedures of the standard General Relativity. The first is to replace T by an arbitrary function $f(T)$ [41, 42, 46–72], similar to $f(R)$ extensions of GR, and obtain new interesting terms in the field equations. The other, on which we focus in the present work, is to add a canonical scalar field in (2.4), similar to the GR quintessence, allowing for a nonminimal coupling between it and gravity. This field will constitute the dark energy sector, and thus the corresponding scenario is called “teleparallel dark energy” [38]. In particular, the action will simply read:

$$S = \int d^4x e \left[\frac{T}{2\kappa^2} + \frac{1}{2} \left(\partial_\mu \phi \partial^\mu \phi + \xi T \phi^2 \right) - V(\phi) + \mathcal{L}_m \right]. \quad (2.5)$$

Concerning the nonminimal coupling we emphasize that the nonminimal coupling will be between the torsion and the scalar field (similar to the standard nonminimal quintessence where the scalar field couples to the Ricci scalar).

Variation of the action (2.5) with respect to the vierbein fields yields the equations of motion

$$\begin{aligned} \left(\frac{2}{\kappa^2} + 2\xi\phi^2 \right) & \left[e^{-1} \partial_\mu (e e_A^\rho S_\rho^{\mu\nu}) - e_A^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} - \frac{1}{4} e_A^\nu T^\mu_{\mu} \right] \\ & - e_A^\nu \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right] + e_A^\mu \partial^\nu \phi \partial_\mu \phi \\ & + 4\xi e_A^\rho S_\rho^{\mu\nu} \phi (\partial_\mu \phi) = e_A^\rho T^{\text{em}}_{\rho}{}^\nu, \end{aligned} \quad (2.6)$$

where $T^{\text{em}}_{\rho}{}^\nu$ stands for the usual energy-momentum tensor. Therefore, for a flat Friedmann-Robertson-Walker (FRW) background metric

$$ds^2 = dt^2 - a^2(t) \delta_{ij} dx^i dx^j \quad (2.7)$$

and a vierbein choice of the form

$$e_\mu^A = \text{diag}(1, a, a, a), \quad (2.8)$$

where t is the cosmic time, x^i are the comoving spatial coordinates and $a(t)$ is the scale factor, we obtain the corresponding Friedmann equations:

$$H^2 = \frac{\kappa^2}{3} (\rho_\phi + \rho_m), \quad (2.9)$$

$$\dot{H} = -\frac{\kappa^2}{2} (\rho_\phi + p_\phi + \rho_m + p_m), \quad (2.10)$$

where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes differentiation with respect to t . In these expressions, ρ_m and p_m are the matter energy density and pressure, respectively, following the standard evolution equation $\dot{\rho}_m + 3H(1 + w_m)\rho_m = 0$, with $w_m = p_m/\rho_m$ the

matter equation-of-state parameter. Additionally, we have introduced the energy density and pressure of the scalar field

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) - 3\xi H^2 \phi^2, \quad (2.11)$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) + 4\xi H \phi \dot{\phi} + \xi (3H^2 + 2\dot{H}) \phi^2. \quad (2.12)$$

Moreover, variation of the action with respect to the scalar field provides its evolution equation, namely:

$$\ddot{\phi} + 3H\dot{\phi} + 6\xi H^2 \phi + V'(\phi) = 0. \quad (2.13)$$

Note that in the above expressions we have used the useful relation $T = -6H^2$, which straightforwardly arises from the calculation of (2.3) for the flat FRW geometry.

In this scenario, similar to the standard quintessence, dark energy is attributed to the scalar field, and thus its equation-of-state parameter (w_{DE}) reads:

$$w_{DE} \equiv w_\phi = \frac{p_\phi}{\rho_\phi}. \quad (2.14)$$

As a result, one can see that the scalar field evolution (2.13) leads to the standard relation

$$\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = 0. \quad (2.15)$$

The teleparallel dark energy proves to exhibit a very interesting cosmological implication [38, 73]. In the minimally-coupled case the cosmological equations coincide with those of the standard quintessence, both at the background and perturbation levels. However, when the nonminimal coupling is switched on, one can obtain a dark energy sector being quintessence-like, phantom-like, or experiencing the phantom-divide crossing during evolution, a behavior that is much richer comparing to General Relativity (GR) with a scalar field [38]. Therefore, it is both interesting and necessary to use observations to constrain the parameters of the scenario. This is performed in the next section.

3 Observational Constraints

We use Type Ia Supernovae (SNIa) from the Supernova Cosmology Project (SCP) Union2 compilation [74], Baryon Acoustic Oscillations (BAO) data from the Two-Degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey Data Release 7 (SDSS DR7) [75], and the Cosmic Microwave Background (CMB) radiation data from Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) observations [76] to examine the teleparallel dark energy scenario. Since the model includes the scalar-field potential, we perform our analysis for three different potential cases, given by:

- Power-Law potential

This potential class is common in cosmology [77–81]. Although we can straightforwardly perform our analysis for an arbitrary exponent, for simplicity we focus on the most interesting quartic case

$$V(\phi) = V_0 \phi^4. \quad (3.1)$$

- Exponential potential

Exponential potentials are also very common in the literature [82–85], which are necessary to be considered in every observational constraining analysis. In the following we use the ansatz

$$V(\phi) = V_0 e^{-\kappa\lambda\phi}. \quad (3.2)$$

- Inverse hyperbolic cosine potential

The use of hyperbolic cosine potential, or power-law functions of it, has also many cosmological implications [86–88]. Although we could perform our analysis for an arbitrary exponent, in the following we focus on the inverse case, namely:

$$V(\phi) = \frac{V_0}{\cosh(\kappa\phi)}. \quad (3.3)$$

We examine the constraints on the model parameters and the present values of the density parameters, following the χ^2 -method for the recent observational data. The detailed analysis method for SNIa, BAO and CMB data is summarized in Appendix. In general, we are interested in producing the likelihood contours for physically-interesting parameters, namely the present dark-energy equation-of-state parameter w_{DE_0} , the present matter density parameter Ω_{m0} and the nonminimal coupling parameter ξ . We mention here that ξ must always be bounded according to a physical constraint, namely it must lead to positive ρ_{DE} and H^2 in relations (2.9) and (2.11), respectively. In practice, ξ is found to be mainly negative (in our convention), and only a small window of positive values is theoretically allowed. In our analysis, for each of the three potentials, we fit three parameters, namely w_{DE_0} , the dimensionless Hubble parameter h and $\Omega_{m0}(\xi)$, and then we draw the likelihood-contours for 1σ and 2σ confidence levels.

In Fig. 1 we present the likelihood contours for w_{DE_0} and Ω_{m0} with the teleparallel dark energy scenario under the quartic potential (3.1). As we observe, the scenario at hand is in agreement with observations, and as expected, it can describe both the quintessence and phantom regimes. Since the scalar field is canonical, it is a great advantage of the present model.

In Fig. 2 we present the likelihood contours for w_{DE_0} and the nonminimal coupling parameter ξ , for the quartic potential (3.1). Interestingly enough we observe that the nonminimal coupling is favored by the data, and in particular a small ξ is responsible for the quintessence regime, while a larger one leads to the phantom regime. Note that the best-fit values of $w_{DE_0}|_{b.f} \approx -0.98$ is very close to the cosmological constant.

In Fig. 3 we present the likelihood contours for w_{DE_0} and Ω_{m0} , for the teleparallel dark energy scenario under the exponential potential (3.2). As we observe, this scenario is consistent with observations, and it can describe both the quintessence and phantom regimes, with the phantom regime favored by the data. Furthermore, in Fig. 4 we present the likelihood contours for w_{DE_0} and ξ , for the exponential potential (3.2). From this graph we deduce that a non-minimal coupling is favored by the data, and we observe that w_{DE_0} -values close to the cosmological constant bound, either above or below it, can be induced

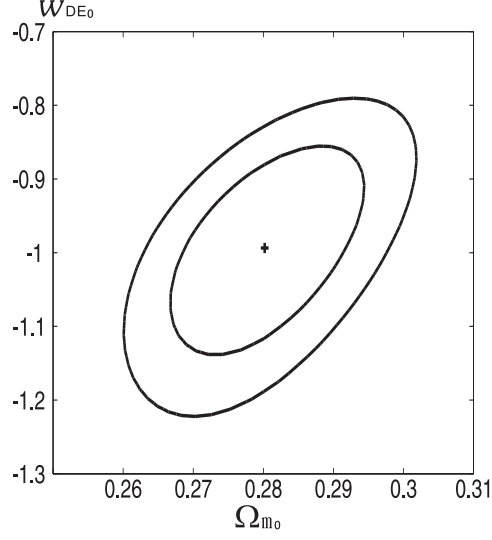


Figure 1. Contour plots of the present dark-energy equation-of-state parameter w_{DE_0} versus the present matter density parameter Ω_{m0} under SNIa, BAO and CMB observational data in the teleparallel dark energy scenario with the quartic potential $V(\phi) = V_0\phi^4$. The curves correspond to 1σ and 2σ confidence levels, respectively, and the cross marks the best-fit point.

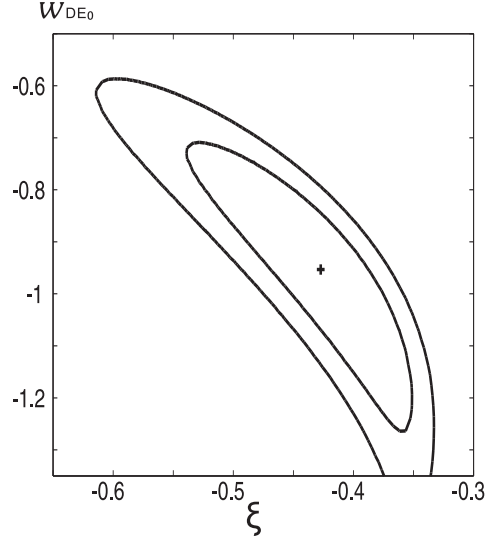


Figure 2. Contour plots of the present dark-energy equation-of-state parameter w_{DE_0} versus the nonminimal coupling parameter ξ under SNIa, BAO and CMB observational data, in the teleparallel dark energy scenario with the quartic potential $V(\phi) = V_0\phi^4$. The curves correspond to 1σ and 2σ confidence levels, respectively, and the cross marks the best-fit point.

by a relative large ξ -interval, which is an advantage of this scenario. It is interesting to mention that the exponential potential was used as an explicit example in [38], and our current analysis verifies its results.

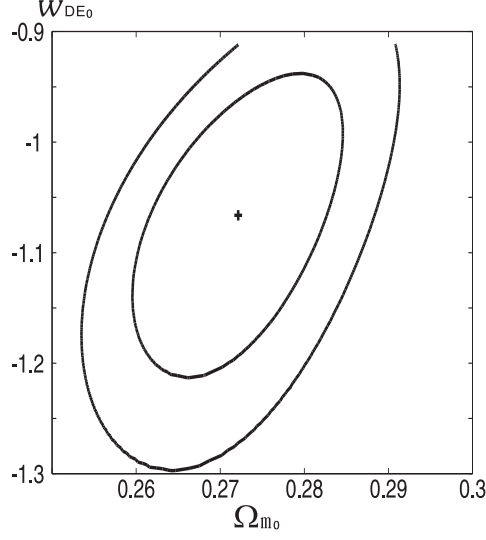


Figure 3. Legend is the same as Fig. 1 but with the exponential potential $V(\phi) = V_0 e^{-\kappa\lambda\phi}$.

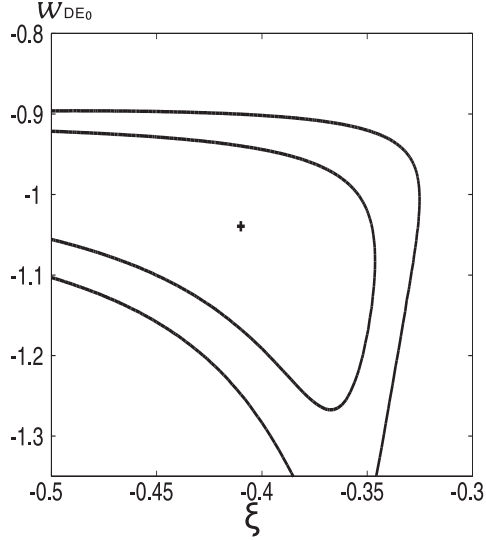


Figure 4. Legend is the same as Fig. 2 but with the exponential potential $V(\phi) = V_0 e^{-\kappa\lambda\phi}$.

In Fig. 5 we depict the likelihood contours for w_{DE_0} and Ω_{m0} , under the inverse hyperbolic cosine potential (3.3). As we can see, this scenario is consistent with observations. However, if we desire to avoid divergences in the w_{DE} evolving history, we are restricted in the phantom regime. In addition, the best-fit value of $w_{DE_0}|_{b.f} \approx -1.02$ is very close to the cosmological constant. Furthermore, in Fig. 6 we present the corresponding likelihood contours for w_{DE_0} and ξ . Similarly to the previous cases we can see that the nonminimal coupling is favored by the data. It is interesting to note that w_{DE_0} -values close to the cosmological constant bound can be induced by a relative large ξ -interval.

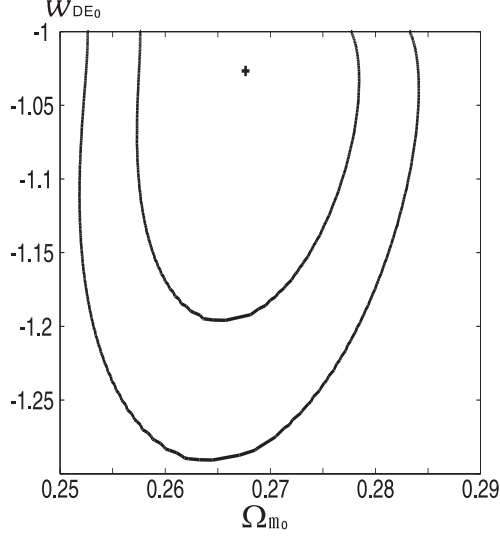


Figure 5. Legend is the same as Fig. 1 but with the inverse hyperbolic cosine potential $V(\phi) = V_0/\cosh(\kappa\phi)$.

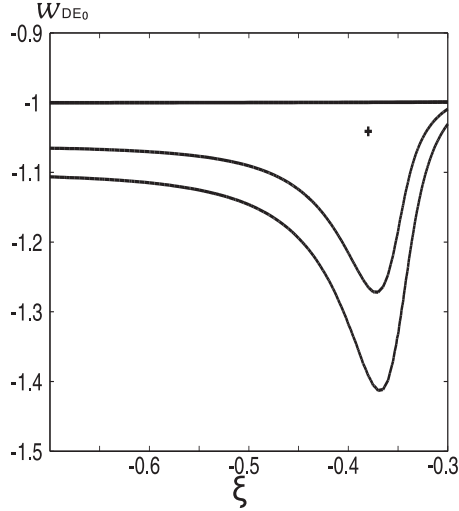


Figure 6. Legend is the same as Fig. 2 but with the inverse hyperbolic cosine potential $V(\phi) = V_0/\cosh(\kappa\phi)$.

We close this section with a comment on the positive values of the nonminimal coupling ξ . As we mentioned above, the positivity requirement for ρ_{DE} and H^2 leads ξ to be negative, with only a small window of positive values theoretically allowed. Now, in practice, if we perform our fitting procedure in the positive ξ region for the inverse hyperbolic cosine potential, we find that positive ξ is excluded. However, for the quartic and exponential potentials we find the interesting result that for the theoretically allowed positive ξ ($0 \leq \xi \lesssim 0.2$) w_{DE} is always very close to a constant w_{DE_0} with $|w_{DE_0} - w_{DE}| \lesssim 10^{-3}$.

The reason is that the scenario of the teleparallel dark energy for positive ξ (sufficiently small in order for the positivity of ρ_{DE} and H^2 not to be spoiled) always results in the stabilization of w_{DE_0} close to the cosmological constant value, as can be proven by a detailed phase-space analysis [89]. Such a behavior is an advantage from both observational and theoretical point of view. We would like to note for comparison that, in the case of quintessence in the Einstein gravity and in the presence of a dust-like component (CDM and baryons), dark energy with an exactly constant $w_{DE} > -1$ is possible for the inverse hyperbolic sine potential in some power as was first independently shown in Refs. [90] and [91].

4 Conclusions

In the present work we have used observational data to impose constraints on the parameters of the teleparallel dark energy scenario [38], which is based on the teleparallel equivalent of General Relativity (TEGR), that is on its torsion instead of curvature formulation [39, 40]. In this model one adds a canonical scalar field, in which the dark energy sector is attributed, allowing also for a nonminimal coupling between the field and the torsion scalar. Thus, although the minimal case is completely equivalent with the standard quintessence, the nonminimal scenario has a richer structure, exhibiting the quintessence-like or phantom-like behavior, or experiencing the phantom-divide crossing [38].

In particular, we have fitted data from Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO), and Cosmic Microwave Background (CMB) observations to constrain the present dark-energy equation-of-state parameter w_{DE_0} , the present matter density parameter Ω_{m0} and the nonminimal coupling parameter ξ . Furthermore, in order to be general, for the scalar-field potential we have taken three ansatzes, namely the power-law, the exponential and the inverse hyperbolic cosine ones.

For the power-law (quartic) potential we have seen that teleparallel dark energy is compatible with observations and, as expected, it can describe both the quintessence and phantom regimes. Additionally, we have shown that the negative nonminimal coupling is favored by the data, and in particular a small ξ is responsible for the quintessence regime, while a larger one leads to the phantom regime. For the exponential potential we have demonstrated that both the quintessence and phantom regimes can be described, with the phantom regime favored by the data. Moreover, we have found that a negative non-minimal coupling is favored and we have observed that w_{DE_0} -values close to the cosmological constant bound, either above or below it, can be induced by a relative large ξ -interval, which is an advantage of this scenario. For the inverse hyperbolic cosine potential we have shown that w_{DE_0} is restricted in the phantom regime, while ξ is restricted to negative values, with a relatively large ξ -interval being able to lead to w_{DE_0} -values close to -1 . We remark that positive values of ξ are excluded for the inverse hyperbolic cosine potential, while for the power-law and exponential ones w_{DE_0} is very close to -1 .

In summary, the scenario of the teleparallel dark energy is compatible with observations, for all the examined scalar-field potentials. Furthermore, although the scalar field is canonical, the model can describe both the quintessence and phantom regimes. These

features are an advantage from both observational and theoretical point of view, and they make the scenario at hand a good candidate for the description of nature. Finally, the data favor a nonminimal coupling, and thus the model is distinguishable from the standard quintessence, since the two scenarios are equivalent only for the minimal coupling.

An interesting and necessary investigation would be to go beyond the background analysis, and examine observables that arise at the perturbation level, such as the growth of matter overdensities and the gravitational-wave spectrum, since these could also clarify possible Lorentz-violation problems that are not seen at the background level [92–94]. Since this study lies beyond the scope of the present manuscript, it is left for future investigation.

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A Analysis method for the observational data

In this appendix, we explain the methods for the elaboration of observational data from Type Ia Supernovae (SNIa), Baryon Acoustic Oscillations (BAO) and Cosmic Microwave Background (CMB) radiation. The χ^2 of the combined observational data is given by

$$\chi^2 = \tilde{\chi}_{\text{SN}}^2 + \chi_{\text{BAO}}^2 + \chi_{\text{CMB}}^2. \quad (\text{A.1})$$

In our fitting procedure we use the simple χ^2 method, rather than the Markov-chain Monte Carlo (MCMC) approach such as CosmoMC [95]. In the following we describe the calculation for the various χ_i^2 of each observational dataset (for detailed explanations on the data analysis see e.g. [96, 97]).

a. Type Ia Supernovae (SNe Ia)

SNe Ia observations provide the information on the luminosity distance D_L as a function of the redshift z . The theoretical distance modulus μ_{th} is defined by

$$\mu_{\text{th}}(z_i) \equiv 5 \log_{10} D_L(z_i) + \mu_0,$$

where $\mu_0 \equiv 42.38 - 5 \log_{10} h$, with $h \equiv H_0/100/[\text{km sec}^{-1} \text{Mpc}^{-1}]$ [76]. The Hubble-free luminosity distance for the flat universe is described as

$$D_L(z) = (1+z) \int_0^z \frac{dz'}{E(z')},$$

where $E(z) \equiv H(z)/H_0$, with

$$\frac{H(z)}{H_0} = \sqrt{\Omega_{\text{m}}^{(0)} (1+z)^3 + \Omega_{\text{r}}^{(0)} (1+z)^4 + \Omega_{\text{DE}}^{(0)} (1+z)^{3(1+w_{\text{DE}})}}.$$

Here, $\Omega_{\text{r}}^{(0)} = \Omega_{\gamma}^{(0)} (1 + 0.2271 N_{\text{eff}})$, where $\Omega_{\gamma}^{(0)}$ is the present fractional photon energy density and $N_{\text{eff}} = 3.04$ is the effective number of neutrino species [76]. We mention that $H(z)$ is evaluated by using numerical solutions of the Friedmann equation.

The χ^2 of the SNe Ia data is given by

$$\chi_{\text{SN}}^2 = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma_i^2}, \quad (\text{A.2})$$

where μ_{obs} is the observed value of the distance modulus. In the following, subscriptions “th” and “obs” denote the theoretically predicted and observed values, respectively. χ_{SN}^2 is minimized with respect to μ_0 , which relates to the absolute magnitude, since the absolute magnitude of SNe Ia is not known. χ_{SN}^2 in (A.2) is expanded as [98]

$$\chi_{\text{SN}}^2 = A - 2\mu_0 B + \mu_0^2 C,$$

with

$$\begin{aligned} A &= \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)]^2}{\sigma_i^2}, \\ B &= \sum_i \frac{\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i; \mu_0 = 0)}{\sigma_i^2}, \\ C &= \sum_i \frac{1}{\sigma_i^2}. \end{aligned}$$

Thus, the minimum of χ_{SN}^2 with respect to μ_0 is expressed as

$$\tilde{\chi}_{\text{SN}}^2 = A - \frac{B^2}{C}. \quad (\text{A.3})$$

In our analysis we apply expression (A.3) for the χ^2 minimization and we use the Supernova Cosmology Project (SCP) Union2 compilation, which contains 557 supernovae [74], ranging from $z = 0.015$ to $z = 1.4$.

b. Baryon Acoustic Oscillations (BAO)

The distance ratio of $d_z \equiv r_s(z_{\text{d}})/D_V(z)$ is measured by BAO observations, where D_V is the volume-averaged distance, r_s is the comoving sound horizon and z_{d} is the redshift at the drag epoch [75]. The volume-averaged distance $D_V(z)$ is defined as [99]

$$D_V(z) \equiv \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3},$$

where $D_A(z)$ is the proper angular diameter distance for the flat universe, defined by

$$D_A(z) \equiv \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}.$$

The comoving sound horizon $r_s(z)$ is given by

$$r_s(z) = \frac{1}{\sqrt{3}} \int_0^{1/(1+z)} \frac{da}{a^2 H(z' = 1/a - 1) \sqrt{1 + \left(3\Omega_b^{(0)}/4\Omega_\gamma^{(0)}\right) a}},$$

where $\Omega_b^{(0)} = 2.2765 \times 10^{-2} h^{-2}$ and $\Omega_\gamma^{(0)} = 2.469 \times 10^{-5} h^{-2}$ are the current values of baryon and photon density parameters, respectively [76]. The fitting formula for z_d is given by [100]

$$z_d = \frac{1291(\Omega_m^{(0)} h^2)^{0.251}}{1 + 0.659(\Omega_m^{(0)} h^2)^{0.828}} \left[1 + b_1 \left(\Omega_b^{(0)} h^2 \right)^{b_2} \right],$$

with

$$b_1 = 0.313(\Omega_m^0 h^2)^{-0.419} \left[1 + 0.607 (\Omega_m^0 h^2)^{0.674} \right],$$

$$b_2 = 0.238 (\Omega_m^0 h^2)^{0.223}.$$

The typical value of z_d is about 1021 for $\Omega_m^{(0)} = 0.276$ and $h = 0.705$.

According to the BAO data from the Two-Degree Field Galaxy Redshift Survey (2dFGRS) and the Sloan Digital Sky Survey Data Release 7 (SDSS DR7) [75], the distance ratio d_z at two redshifts $z = 0.2$ and $z = 0.35$ is measured to be $d_{z=0.2}^{\text{obs}} = 0.1905 \pm 0.0061$ and $d_{z=0.35}^{\text{obs}} = 0.1097 \pm 0.0036$, with the inverse covariance matrix:

$$C_{\text{BAO}}^{-1} = \begin{pmatrix} 30124 & -17227 \\ -17227 & 86977 \end{pmatrix}.$$

Finally, the χ^2 for the BAO data is calculated as

$$\chi_{\text{BAO}}^2 = \left(x_{i,\text{BAO}}^{\text{th}} - x_{i,\text{BAO}}^{\text{obs}} \right) (C_{\text{BAO}}^{-1})_{ij} \left(x_{j,\text{BAO}}^{\text{th}} - x_{j,\text{BAO}}^{\text{obs}} \right),$$

where $x_{i,\text{BAO}} \equiv (d_{0.2}, d_{0.35})$.

c. Cosmic Microwave Background (CMB) radiation

The CMB observational data are sensitive to the distance to the decoupling epoch z_* [76]. Hence, by using these data we obtain constraints on the model in the high redshift regime ($z \sim 1000$).

The acoustic scale l_A and the shift parameter \mathcal{R} [101] are defined by

$$l_A(z_*) \equiv (1 + z_*) \frac{\pi D_A(z_*)}{r_s(z_*)},$$

$$\mathcal{R}(z_*) \equiv \sqrt{\Omega_m^{(0)}} H_0 (1 + z_*) D_A(z_*),$$

where z_* is the redshift of the decoupling epoch, given by [102]

$$z_* = 1048 \left[1 + 0.00124 \left(\Omega_b^{(0)} h^2 \right)^{-0.738} \right] \left[1 + g_1 \left(\Omega_m^{(0)} h^2 \right)^{g_2} \right],$$

with

$$g_1 = \frac{0.0783 \left(\Omega_b^{(0)} h^2 \right)^{-0.238}}{1 + 39.5 \left(\Omega_b^{(0)} h^2 \right)^{0.763}}, \quad g_2 = \frac{0.560}{1 + 21.1 \left(\Omega_b^{(0)} h^2 \right)^{1.81}}.$$

We use the data from Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) observations [76] on CMB.

The χ^2 of the CMB data is

$$\chi_{\text{CMB}}^2 = \left(x_{i,\text{CMB}}^{\text{th}} - x_{i,\text{CMB}}^{\text{obs}} \right) (C_{\text{CMB}}^{-1})_{ij} \left(x_{j,\text{CMB}}^{\text{th}} - x_{j,\text{CMB}}^{\text{obs}} \right),$$

where $x_{i,\text{CMB}} \equiv (l_A(z_*), \mathcal{R}(z_*), z_*)$ and C_{CMB}^{-1} is the inverse covariance matrix. The data from WMAP7 observations [76] lead to $l_A(z_*) = 302.09$, $\mathcal{R}(z_*) = 1.725$ and $z_* = 1091.3$ with the inverse covariance matrix:

$$C_{\text{CMB}}^{-1} = \begin{pmatrix} 2.305 & 29.698 & -1.333 \\ 29.698 & 6825.27 & -113.180 \\ -1.333 & -113.180 & 3.414 \end{pmatrix}.$$

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